repertories of magical rituals from any part of the world. For certain departments of ancient magic, however, like the Pythagorean philosophy, there is no lack of illustrative material; it depended on mystical speculations based on numbers or analogous principles. The importance of numbers is recognized in the magic of America and other areas, but the science of the Mediterranean area, combined with the art of writing, was needed to develop such mystical ideas to their full extent. Among the neo-Platonists there was a strong tendency to magical speculation, and they sought to impress into their service the demons with which they peopled the universe. Alexandria was the home of many systems of theurgic magic, and gnostic gems afford evidence of the nature of their symbols. In the middle ages the respectable branches of magic, such as astrology and alchemy, included much of the real science of the period; the rise of Christianity introduced a new element, for the Church regarded all the religions of the heathen as dealings with demons and therefore magical (see WITCHCRAFT). In our own day the occult sciences still find devotees among the educated; certain elements have acquired a new interest, in so far as they are the subject matter of psychical research (*q.v.*) and spiritualism (*q.v.*). But it is only among what are regarded as the lower classes, and in England especially the rural population, that belief in its efficacy still prevails to any large extent.

Psychology of Magic.—The same causes which operated to produce a belief in witchcraft (*q.v.*) aided the creed of magic in general. Fortuitous coincidences attract attention; the failures are disregarded or explained away. Probably the magician is never wholly an impostor, and frequently has a whole-hearted belief in himself; in this connexion may be noted the fact that juggling tricks have in all ages been passed off as magical; the name of "conjuring" (*q.v.*) survives in our own day, though the conjurer no longer claims that his mysterious results are produced by demons. It is interesting to note that magical leechcraft depended for its success on the power of suggestion (*q.v.*), which is to-day a recognized element in medicine; perhaps other elements may have been instrumental in producing a cure, for there are cases on record in which European patients have been cured by the apparently meaningless performances of medicine-men, but an adequate study of savage medicine is still a desideratum.

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(N. W. T.)

MAGIC SQUARE, a square divided into equal squares, like a chess-board, in each of which is placed one of a series of consecutive numbers from 1 up to the square of the number of cells in a side, in such a manner that the sum of the numbers in each row or column and in each diagonal is constant.

From a very early period these squares engaged the attention of mathematicians, especially such as possessed a love of the marvellous, or sought to win for themselves a superstitious regard. They were then supposed to possess magical properties, and were worn, as in India at the present day, engraven in metal or stone, as amulets or talismans. According to the old astrologers, relations subsisted between these squares and the planets. In later times such squares ranked only as mathematical curiosities; till at last their mode of construction was systematically investigated. The earliest known writer on the subject was Emanuel Moscopulus, a Greek (4th or 5th century). Bernard Frenicle de Bessy constructed magic squares such that if one or more of the encircling bands of numbers be taken away the remaining central squares are still magical. Subsequently Poignard constructed squares with numbers in arithmetical progression, having the magical summations. The later researches of Phillipe de la Hire, recorded in the *Mémoires de l'Académie Royale* in 1705, are interesting as giving general methods of construction. He has there collected the results of the labours of earlier pioneers; but the subject has now been fully systematized, and extended to cubes.

200	217	232	249	8	25	40	57	72	89	104	121	136	153	168	185
58	39	26	7	250	231	218	199	186	167	154	135	122	103	90	71
198	219	230	251	6	27	38	59	70	91	102	123	134	155	166	187
60	37	28	5	252	229	220	197	188	165	156	133	124	101	92	69
201	216	233	248	9	24	41	56	73	88	105	120	137	152	169	184
55	42	23	10	247	234	215	202	183	170	151	138	119	106	87	74
203	214	235	246	11	22	43	54	75	86	107	118	139	150	171	182
53	44	21	12	245	236	213	204	181	172	149	140	117	108	85	76
205	212	237	244	13	20	45	52	77	84	109	116	141	148	173	180
51	46	19	14	243	238	211	206	179	174	147	142	115	110	83	78
207	210	239	242	15	18	47	50	79	82	111	114	143	146	175	178
49	48	17	16	241	240	209	208	177	176	145	144	113	112	81	80
196	221	228	253	4	29	36	61	68	93	100	125	132	157	164	189
62	35	30	3	254	227	222	195	190	163	158	131	126	99	94	67
194	223	226	255	2	31	34	63	66	95	98	127	130	159	162	191
64	33	32	1	256	225	224	193	192	161	160	129	128	97	96	65

FIG. 1.

Two interesting magical arrangements are said to have been given by Benjamin Franklin; these have been termed the "magic square of squares" and the "magic circle of circles." The first (fig. 1) is a square divided into 256 squares, *i.e.* 16 squares along a side, in which are placed the numbers from 1 to 256. The chief properties of this square are (1) the sum of the 16 numbers in any row or column is 2056; (2) the sum of the 8 numbers in half of any row or column is 1028, *i.e.* one half of 2056; (3) the sum of the numbers in two half-diagonals equals 2056; (4) the sum of the four corner numbers of the great square and the four central numbers equals 1028; (5) the sum of the numbers in any 16 cells of the large square which themselves are disposed in a square is 2056. This square has other curious

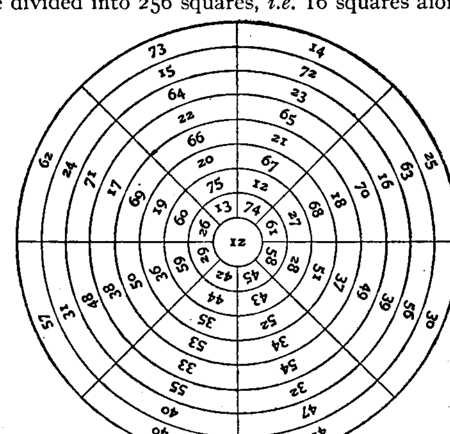

FIG. 2.

properties. The "magic circle of circles" (fig. 2) consists of eight annular rings and a central circle, each ring being divided into eight cells by radii drawn from the centre; there are therefore 65 cells. The number 12 is placed in the centre, and the consecutive numbers 13 to 75 are placed in the other cells. The properties of this figure include the following: (1) the sum of the eight numbers in any ring together with the central number 12 is 360, the number of degrees in a circle; (2) the sum of the eight numbers in any set of radial cells together with the central number is 360; (3) the sum of the numbers in any four adjoining cells, either annular, radial, or both radial and two annular, together with half the central number, is 180.

Construction of Magic Squares.—A square of 5 (fig. 3) has adjoining it one of the eight equal squares by which any square may be conceived to be surrounded, each of which has two sides resting on adjoining squares, while four have sides resting on the surrounded square, and four meet it only at its four angles.

	a	ε	5	
	4	*b*		δ
	γ		*c*	3
		2	β	*d*
1*e* *a*				

		ε		
	4			δ
	γ			
e				

FIG. 3.

1, 2, 3 are placed along the path of a knight in chess; 4, along the same path, would fall in a cell of the outer square, and is placed instead in the corresponding cell of the original square; 5 then falls within the square. *a*, *b*, *c*, *d* are placed diagonally in the square; but *e* enters the outer square, and is removed thence to the same cell of the square it had left. α, β, γ, δ, ε pursue another regular course; and the diagram shows how that course is recorded in the square they have twice left. Whichever of the eight surrounding squares may be entered, the corresponding cell of the central square is taken instead. The 1, 2, 3, ..., *a*, *b*, *c*, ..., α, β, γ, ... are said to lie in "paths."

Squares whose Roots are Odd.—Figs 4, 5, and 6 exhibit one of the earliest methods of constructing magic squares. Here the

1	4	2	5	3
4	2	5	3	1
2	5	3	1	4
5	3	1	4	2
3	1	4	2	5

FIG. 4.

2	4	6	3	1
1	2	4	0	3
3	1	2	4	0
0	3	1	2	4
4	0	3	1	2

FIG. 5.

11	24	2	20	8
9	12	25	3	16
17	10	13	21	4
5	18	6	14	22
23	1	19	7	15

FIG. 6.

3's in fig. 4 and 2's in fig. 5 are placed in opposite diagonals to secure the two diagonal summations; then each number in fig. 5 is multiplied by 5 and added to that in the corresponding square in fig. 4, which gives the square of fig. 6. Figs. 7, 8 and 9 give De la Hire's method; the squares of figs. 7 and 8, being combined, give the magic square of fig. 9. C. G. Bachet arranged the numbers as in fig. 10, where there are three numbers in each of four surrounding squares; these being placed in the corresponding cells of the central square, the square of fig. 11 is formed. He also con-

2	1	5	3	4
3	4	2	1	5
1	5	3	4	2
4	2	1	5	3
5	3	4	2	1

FIG. 7.

15	5	0	20	10
0	20	10	15	5
10	15	5	0	20
5	0	20	10	15
20	10	15	5	0

FIG. 8.

17	6	5	23	14
3	24	12	16	10
11	20	8	4	22
9	2	21	15	18
25	13	19	7	1

FIG. 9.

structed squares such that if one or more outer bands of numbers are removed the remaining central squares are magical. His method of forming them may be understood from a square of 5. Here each summation is 5×13; if therefore 13 is subtracted from each number, the summations will be zero, and the twenty-five cells will contain the series ± 1, ± 2, ± 3, ... ± 12, the odd cell having 0. The central square of 3 is formed with four of the twelve numbers with + and − signs and zero in the middle; the band is filled up with the rest, as in fig. 12; then, 13 being added in each cell, the magic square of fig. 13 is obtained.

Squares whose Roots are Even.—These were constructed in various ways, similar to that of 4 in figs. 14, 15 and 16. The numbers in fig. 15 being multiplied by 4, and the squares of figs. 14 and 15 being superimposed, give fig. 16. The application of

				1				
			6		2			
		11		7		3		
	16		12		8		4	
21		17		13		9		5
	22		18		14		10	
		23		19		15		
			24		20			
				25				

FIG. 10.

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

FIG. 11.

this method to squares the half of whose roots are odd requires a complicated adjustment. Squares whose half root is a multiple of 4, and in which there are summations along all the diagonal

-9	12	5	-2	-6
1	7	-11	4	-1
-8	-3	0	3	8
10	-4	11	-7	-10
6	-12	-5	2	9

FIG. 12.

4	25	18	11	7
14	20	2	17	12
5	10	13	16	21
23	9	24	6	3
19	1	8	15	22

FIG. 13.

paths, may be formed, by observing, as when the root is 4, that the series 1 to 16 may be changed into the series 15, 13, ... 3, 1, −1, −3, ... −13, −15, by multiplying each number by 2

1	3	2	4
4	2	3	1
4	2	3	1
1	3	2	4

FIG. 14.

0	3	3	0
2	1	1	2
1	2	2	1
3	0	0	3

FIG. 15.

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

FIG. 16.

and subtracting 17; and, vice versa, by adding 17 to each of the latter, and dividing by 2. The diagonal summations of a square, filled as in fig. 17, make zero; and, to obtain the same in the rows

p_1	p_2	a_1	a_2
p_3	p_4	a_3	a_4
$-a_1$	$-a_2$	$-p_1$	$-p_2$
$-a_3$	$-a_4$	$-p_3$	$-p_4$

FIG. 17.

1	-3	11	-9
-5	7	-15	13
-11	9	-1	3
15	-13	5	-7

FIG. 18.

9	7	14	4
6	12	1	15
3	13	8	10
16	2	11	5

FIG. 19.

and columns, we must assign such values to the p's and q's as satisfy the equations $p_1 + p_2 + a_1 + a_2 = 0$, $p_3 + p_4 + a_3 + a_4 = 0$, $p_1 + p_3 - a_1 - a_3 = 0$, and $p_2 + p_4 - a_2 - a_4 = 0$,—a solution of which is readily obtained by inspection, as in fig. 18; this leads to the square, fig. 19. When the root is 8, the upper four subsidiary rows may at once be written, as in fig, 20; then, if 65 be added to each, and the sums halved, the square is completed. In such squares as these, the two opposite squares about the same diagonal (except that of 4) may be turned through any number of right angles, in the same direction, without altering the summations.

-1	3	5	-7	-33	35	37	-39
9	-11	-13	15	41	-43	-45	47
17	-19	-21	23	49	-51	-53	55
-25	27	29	-31	-57	59	61	-63

FIG. 20.

Nasik Squares.—Squares that have many more summations than in rows, columns and diagonals were investigated by A. H. Frost (*Cambridge Math. Jour.*, 1857), and called Nasik squares, from the town in India where he resided; and he extended the method to cubes, various sections of which have the same singular properties. In order to understand their construction it will be necessary to consider carefully fig. 21, which shows that, when the root is a prime, and not composite, number, as 7, eight letters $a, b, \dots h$ may proceed from any, the same, cell, suppose that marked 0, each letter being repeated in the cells along different paths. These eight paths are called "normal paths," their number being one more than the root. Observe here that, excepting the cells from which any two letters start, they do not occupy again the same cell, and that two letters, starting from any two different cells along different paths, will appear together in one and only one cell. Hence, if p_1 be placed in the cells of one of the $n+1$ normal paths, each of the remaining n normal paths will contain one, and only one, of these p_1's. If now we fill each row with $p_2, p_3, \dots p_n$ in the same order, commencing from the p_1 in that row, the p_2's, p_3's and p_n's will lie each in a path similar to that of p_1, and each of the n normal paths will contain one, and only one, of the letters $p_1, p_2, \dots p_n$, whose sum will be Σp. Similarly, if q_1 be placed along any of the normal paths, different from that of the p's, and each row filled as above with the letters $q_2, q_3, \dots q_n$, the sum of the q's along any normal path different from that of the q_1 will be Σq. The n^2 cells of the square will now be found to contain all the combinations of the p's and q's; and if the q's be multiplied by n, the p's made equal to 1, 2, ... n, and the q's to 0, 1, 2, ... $(n-1)$ in any order, the Nasik square of n will be obtained, and the summations along all the normal paths, except those traversed by the p's and q's, will be the constant $\Sigma nq + \Sigma p$.

a	*g*	*f*	*e*	*d*	*c*	*b*
a	*d*	*g*	*c*	*f*	*b*	*e*
a	*c*	*e*	*g*	*b*	*d*	*f*
a	*f*	*d*	*b*	*g*	*e*	*c*
a	*e*	*b*	*f*	*c*	*g*	*d*
a	*b*	*c*	*d*	*e*	*f*	*g*
0	*h*	*h*	*h*	*h*	*h*	*h*

FIG. 21.

When the root is an odd composite number, as 9, 15, &c., it will be found that in some paths, different from the two along which the p_1 and q_1 were placed, instead of having each of the p's and q's, some will be wanting, while some are repeated. Thus, in the case of 9, the triplets, $p_1p_4p_7$, $p_2p_5p_8$, $p_3p_6p_9$, and $q_1q_4q_7$, $q_2q_5q_8$, $q_3q_6q_9$ occur, each triplet thrice, along paths whose summation should be—Σp 45 and Σr 36. But if we make $p_1, p_2, \dots p_9 =$ 1, 3, 6, 5, 4, 7, 9, 8, 2, and the $r_1, r_2, \dots r_9 =$ 0, 2, 5, 4, 3, 6, 8, 7, 1, thrice each of the above sets of triplets will equal Σp and Σq respectively. If now the q's are multiplied by 9, and added to the p's in their several cells, we shall have a Nasik square, with a constant summation along eight of its ten normal paths. In fig. 22 the numbers are in the nonary scale; that in the centre is the middle one of 1 to 9^2, and the sum of pair of numbers equidistant from and opposite to the central 45 is twice 45; and the sum of any number and the 8 numbers 3 from it, diagonally, and in its row and column, is the constant Nasical summation, *e.g.* 72 and 32, 22, 76, 77, 26, 37, 36, 27.

63	88	74	13	8	24	53	48	34
11	9	25	51	49	35	61	89	75
52	47	36	62	87	76	12	7	26
68	84	73	18	4	23	58	44	33
19	5	21	59	45	31	69	85	71
57	46	32	67	86	72	17	6	22
64	83	78	14	3	28	54	43	38
15	1	29	55	41	39	65	81	79
56	42	37	66	82	77	16	2	27

FIG. 22.

The numbers in fig. 22 being kept in the nonary scale, it is not necessary to add any nine of them together in order to test the Nasical summation; for, taking the first column, the figures in the place of units are seen at once to form the series, 1, 2, 3, ... 9, and those in the other place three triplets of 6, 1, 5. For the squares of 15 the p's and q's may be respectively 1, 2, 10, 8, 6, 14, 15, 11, 4, 13, 9, 7, 3, 12, 5, and 0, 1, 9, 7, 5, 13, 14, 10, 3, 12, 8, 6, 2, 11, 4, where five times the sum of every third number and three times the sum of every fifth number makes Σp and Σq; then, if the q's are multiplied by 15, and added to the p's, the Nasik square of 15 is obtained. When the root is the multiple of 4, the same process gives us, for the square of 4, fig. 23. Here the columns give Σp, but alternately $2q_1$, $2q_3$, and $2q_2$, $2q_4$; and the rows give Σq, but alternately $2p_1$, $2p_3$, and $2p_2$, $2p_4$; the diagonals giving Σp and Σq.

p_4q_3	p_2q_4	p_4q_1	p_2q_2
p_3q_1	p_1q_2	p_3q_3	p_1q_4
p_2q_3	p_4q_4	p_2q_1	p_4q_2
p_1q_1	p_3q_2	p_1q_3	p_3q_4

FIG. 23.

15	10	3	6
4	5	16	9
14	11	2	7
1	8	13	12

FIG. 24.

If p_1, p_2, p_3, p_4 and q_1, q_2, q_3, q_4 be 1, 2, 4, 3, and 0, 1, 3, 2, we have the Nasik square of fig. 24. A square like this is engraved in the Sanskrit character on the gate of the fort of Gwalior, in India. The squares of higher multiples of 4 are readily obtained by a similar adjustment.

Nasik Cubes.—A Nasik cube is composed of n^3 small equal cubes, here called cubelets, in the centres of which the natural numbers from 1 to n^3 are so placed that every section of the cube by planes perpendicular to an edge has the properties of a Nasik square; also sections by planes perpendicular to a face, and passing through the cubelet centres of any path of Nasical summation in that face. Fig. 25 shows by dots the way in which these cubes are constructed. A dot is here placed on three faces of a cubelet at the corner, showing that this cubelet belongs to each of the faces AOB, BOC, COA, of the cube. Dots are placed on the cubelets of some path of AOB (here the knight's path), beginning from O, also on the cubelets of a knight's path in BOC. Dots are now placed in the cubelets of similar paths to that on BOC in the other six sections parallel to BOC, starting from their dots in AOB. Forty-nine of the three hundred and forty-three cubelets will now contain a dot; and it will be observed that the dots in sections perpendicular to BO have arranged themselves in similar paths. In this manner, p_1, q_1, r_1 being placed in the corner cubelet O, these letters are severally placed in the cubelets of three different paths of AOB, and again along any similar paths in the seven sections perpendicular to AO, starting from the letters' position in AOB. Next, $p_2q_2r_2$, $p_2q_3r_3$, ... $p_7q_7r_7$ are placed in the other cubelets of the edge AO, and dispersed in the same manner as $p_1q_1r_1$. Every cubelet will then be found to contain a different combination of the p's, q's and r's. If therefore the p's are made equal to 1, 2, ... 7, and the q's and r's to 0, 1, 2, ... 6, in any order, and the q's multiplied by 7, and the r's by 7^2, then, as in the case of the squares, the 7^3 cubelets will contain the numbers from 1 to 7^3, and the Nasical summations will be $\Sigma 7^2 r + \Sigma 7q + p$. If 2, 4, 5 be values of r, p, q, the number for that cubelet is written 245 in the septenary scale, and if all the cubelet numbers are kept thus, the paths along which summations are found can be seen without adding, as the seven numbers would contain 1, 2, 3, ... 7 in the unit place, and 0, 1, 2, ... 6 in each of the other places. In all Nasik cubes, if such values are given to the letters on the central cubelet that the number is the middle one of the series 1 to n^3, the sum of all the pairs of numbers opposite to and equidistant from the middle number is the double of it.

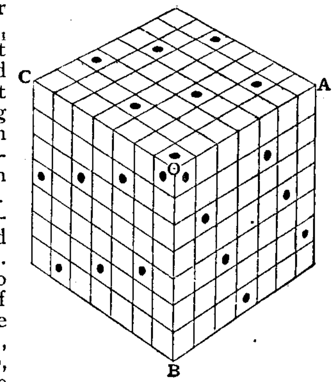

FIG. 25—Nasik Cube.

1	8	29	28		11	14	23	18
30	27	2	7		21	20	9	16
4	5	32	25		10	15	22	19
31	26	3	6		24	17	12	13

FIG. 26.

Also, if around a Nasik cube the twenty-six surrounding equal cubes be placed with their cells filled with the same numbers, and their corresponding faces looking the same way,—and if the surrounding space be conceived thus filled with similar cubes, and a straight line of unlimited length be drawn through any two cubelet centres, one in each of any two cubes,—the numbers along that line will be found to recur in groups of seven, which (except in the three cases where the same p, q or r recur in the group) together make the Nasical summation of the cube. Further, if we take n similarly filled Nasik cubes of n, n new letters, $s_1, s_2, \dots s_n$, can be so placed, one in each of the n^4 cubelets of this group of n cubes, that each shall contain a different combination of the p's, q's, r's and s's. This is done by placing s_1 on each of the n^2 cubelets of the first cube that contain p_1, and on the n^2 cubelets of the 2d, 3d, ... and nth cube that contain $p_2, p_3, \dots p_n$ respectively. This process is repeated with s_2, beginning with the cube at which we ended, and so on with the other s's; the n^4 cubelets, after multiplying the q's, r's, and s's by n, n^2, and n^3 respectively, will now be filled with the numbers from 1 to n^4, and the constant summation will be $\Sigma n^3 s + \Sigma n^2 r + \Sigma nq + \Sigma p$. This process may be carried on without limit; for, if the n cubes are placed in a row with their faces resting on each other, and the corresponding faces looking the same way, n such parallelepipeds might be put side by side, and the n^5 cubelets of this solid square be Nasically filled by the introduction of a new letter t; while, by introducing another letter, the n^6 cubelets of the compound cube of n^3 Nasik

23	18	11	6	25
10	5	24	17	12
19	22	13	4	7
14	9	2	21	16
1	20	15	8	3

FIG. 27.

30	21	6	15	28	19
7	16	29	20	5	14
22	31	8	35	18	27
9	36	17	26	13	4
32	23	2	11	34	25
1	10	33	24	3	12

FIG. 28.

cubes might be filled by the numbers from 1 to n^6, and so *ad infinitum*. When the root is an odd composite number the values of the three groups of letters have to be adjusted as in squares, also in cubes of an even root. A similar process enables us to place successive numbers in the cells of several equal squares in which the Nasical summations are the same in each, as in fig. 26.

Among the many ingenious squares given by various writers, this article may justly close with two by L. Euler, in the *Histoire de l'académie royale des sciences* (Berlin, 1759). In fig. 27 the natural numbers show the path of a knight that moves within an odd square in such a manner that the sum of pairs of numbers opposite to and equidistant from the middle figure is its double. In fig. 28 the knight returns to its starting cell in a square of 6, and the difference between the pairs of numbers opposite to and equidistant from the middle point is 18.

A model consisting of seven Nasik cubes, constructed by A. H. Frost, is in the South Kensington Museum. The centres of the cubes are placed at equal distances in a straight line, the similar faces looking the same way in a plane parallel to that line. Each of the cubes has seven parallel glass plates, to which, on one side, the seven numbers in the septenary scale are fixed, and behind each, on the other side, its value in the common scale. 1201, the middle number from 1 to 7^4, occupies the central cubelet of the middle cube. Besides each cube having separately the same Nasical summation, this is also obtained by adding the numbers in any seven similarly situated cubelets, one in each cube. Also, the sum of all pairs of numbers, in a straight line, through the central cube of the system, equidistant from it, in whatever cubes they are, is twice 1201. (A. H. F.)

Fennell's Magic Ring.—It has been noticed that the numbers of magic squares, of which the extension by repeating the rows and columns of n numbers so as to form a square of $2n-1$ sides yields n^2 magic squares of n sides, are arranged as if they were all inscribed round a cylinder and also all inscribed on another cylinder at right angles to the first. C. A. M. Fennell explains this apparent anomaly by describing such magic squares as Mercator's projections, so to say, of "magic rings."

The surface of these magic rings is symmetrically divided into n^2 quadrangular compartments or cells by n equidistant zonal circles parallel to the circular axis of the ring and by n transverse circles which divide each of the n zones between any two neighbouring zonal circles into n equal quadrangular cells, while the zonal circles divide the sections between two neighbouring transverse circles into n unequal quadrangular cells. The diagonals of cells which follow each other passing once only through each zone and section, form similar and equal closed curves passing once quite round the circular axis of the ring and once quite round the centre of the ring. The position of each number is regarded as the intersection of two diagonals of its cell. The numbers are most easily seen if the smallest circle on the surface of the ring, which circle is concentric with the axis, be one of the zonal circles. In a perfect magic ring the sum of the numbers of the cells whose diagonals form any one of the $2n$ diagonal curves aforesaid is $\frac{1}{2}n(n^2+1)$ with or without increment, *i.e.* is the same sum as that of the numbers in each zone and each transverse section. But if n be 3 or a multiple of 3, only from 2 to n of the diagonal curves carry the sum in question, so that the magic rings are imperfect; and any set of numbers which can be arranged to make a perfect magic ring or magic square can also make an imperfect magic ring, *e.g.* the set 1 to 16 if the numbers 1, 6, 11, 16 lie thus on a diagonal curve instead of in the order 1, 6, 16 11. From a perfect magic ring of n^2 cells containing one number each, n^2 distinct magic squares can be read off; as the four numbers round each intersection of a zonal circle and a transverse circle constitute corner numbers of a magic square. The shape of a magic ring gives it the function of an indefinite extension in all directions of each of the aforesaid n^2 magic squares. (C. A. M. F.)

See F. E. A. Lucas, *Récréations mathématiques* (1891–1894); W.W.R. Ball, *Mathematical Recreations* (1892); W. E. M. G. Ahrens, *Mathematische Unterhaltungen und Spiele* (1901); H. C. H. Schubert, *Mathematische Mussestunden* (1900). A very detailed work is B. Violle, *Traité complet des carrés magiques* (3 vols., 1837–1838). The theory of "path nasiks" is dealt with in a pamphlet by C. Planck (1906).

MAGINN, WILLIAM (1793–1842), Irish poet and journalist, was born at Cork on the 10th of July 1793. The son of a schoolmaster, he graduated at Trinity College, Dublin, in 1811, and after his father's death in 1813 succeeded him in the school. In 1819 he began to contribute to the *Literary Gazette* and to *Blackwood's Magazine*, writing as "R. T. Scott" and "Morgan O'Doherty." He first made his mark as a parodist and a writer of humorous Latin verse. In 1821 he visited Edinburgh, where he made acquaintance with the Blackwood circle. He is credited with having originated the idea of the *Noctes ambrosianae*, of which some of the most brilliant chapters were his. His connexion with Blackwood lasted, with a short interval, almost to the end of his life. His best story was "Bob Burke's Duel with Ensign Brady." In 1823 he removed to London. He was employed by John Murray on the short-lived *Representative*, and was for a short time joint-editor of the *Standard*. But his intemperate habits and his imperfect journalistic morality prevented any permanent success. In connexion with Hugh Fraser he established *Fraser's Magazine* (1830), in which appeared his "Homeric Ballads." Maginn was the original of Captain Shandon in *Pendennis*. In spite of his inexhaustible wit and brilliant scholarship, most of his friends were eventually alienated by his obvious failings and his persistent insolvency. He died at Walton-on-Thames on the 21st of August 1842.

His *Miscellanies* were edited (5 vols., New York, 1855–1857) by R. Shelton Mackenzie and (2 vols., London, 1885) by R. W. Montagu [Johnson].

MAGISTRATE (Lat. *magistratus*, from *magister*, master, properly a public office, hence the person holding such an office), in general, one vested with authority to administer the law or one possessing large judicial or executive authority. In this broad sense the word is used in such phrases as "the first magistrate" of a king in a monarchy or "the chief magistrate" of the president of the United States. But it is more generally applied to minor or subordinate judicial officers, whether unpaid, as justices of the peace, or paid, as stipendiary magistrates. A stipendiary magistrate is appointed in London under the Metropolitan Police Courts Act 1839, in municipal boroughs under the Municipal Corporations Act 1882, and in particular districts under the Stipendiary Magistrates Act 1863 and special acts. In London and municipal boroughs a stipendiary magistrate must be a barrister of at least seven years' standing, while under the Stipendiary Magistrates Act 1863 he may be of five years' standing. A stipendiary magistrate may do alone all acts authorized to be done by two justices of the peace.

The term *magistratus* in ancient Rome originally implied the office of *magister* (master) of the Roman people, but was subsequently applied also to the holder of the office, thus becoming identical in sense with *magister*, and supplanting it in reference to any kind of public office. The fundamental conception of Roman magistracy is tenure of the *imperium*, the sovereignty which resides with the Roman people, but is by it conferred either upon a single ruler for life, as in the later monarchy, or upon a college of magistrates for a fixed term, as in the Republican period. The Roman theory of magistracy underwent little change when two consuls were substituted for the king; but the subdivision of magisterial powers which characterized the first centuries of the Republic, and resulted in the establishment of twenty annually elected magistrates of the people, implied some modification of this principle of the investiture of magistrates with supreme authority. For when the magistracies were multiplied a distinction was drawn between magistrates with *imperium*, namely consuls, praetors and occasionally dictators, and the remaining magistrates, who, although exercising independent magisterial authority and in no sense agents of the higher magistrates, were invested merely with an authority (*potestas*) to assist in the administration of the state. At the same time the actual authority of every magistrate was weakened not only by his colleagues' power of veto, but by the power possessed by any magistrate of quashing the act of an inferior, and by the tribune's right of putting his veto on the act of any magistrate except a dictator; and the subdivision of authority, which placed a great deal of business in the hands of young and inexperienced magistrates, further tended to increase the actual power as well as the influence of the senate at the expense of the magistracy.

In the developed Republic magistracies were divided into two classes: (*a*) magistrates of the whole people (*populi Romani*) and (*b*) magistrates of the *plebs*. The former class is again divided into two sections: (α) curule and (β) non-curule, a distinction which rests mainly on dignity rather than on actual power, for it cuts across the division of magistrates according to their tenure or non-tenure of *imperium*.